Chapter V. *Containing the Doctrine of Spheric Triangles, Rectangular, and Oblique, both Geometric, Logarithmic, and Instrumental.*

BEFORE I enter on Spheric Trigonometry, as to the framing and working of Proportions therein, it will be necessary you should understand how to make a Spheric Triangle, and to measure any of its Parts: In order thereunto I have contrived the following Problems, which I call Spheric Geometry.

Section I. Spheric Geometry *explained by Definitions and Problems.*

Definition 1. *SPheric Geometry*, is that by which the Circles of the Sphere are described, drawn, or projected on a Plane, or flat Superficies.

2. A Sphere or Globe, is a round Body, made by the moving of a Semi-Circle about its own Diameter, till the Motion end where it began.

3. The Projection of the Sphere, is either Orthographic, Stereographic, or Gnomonic.

4. Orthographic is the drawing the Superficies of the Sphere on a Plane, which cutteth the Sphere in the middle, with respect to the Eye being placed perpendicular to it, and at an infinite Distance therefrom: This Projection maketh Use only of the Lines of Chords, and Sines.

5. Stereographic, sheweth how to describe the Sphere's Superficies on a Plane, which cutteth it in the middle, with respect to the Eye being placed in the Sphere's Superficies, perpendicular to the Center of the said Plane.

6. Gnomonic Projection of the Sphere, is drawing the Superficies on a Plane touching it, with respect to the Eye being placed on the Sphere's Center.

These two last require the use of the Lines of Chords, Tangents, and Secants.

7. All Circles of the Sphere, are either Great Circles, which cut the Sphere into two equal Parts; or lesser Circles, which divide it into two unequal Parts.

8. The

8. The Plane on which the Sphere is projected, is that Circle which bounds, or limits the Projection, and is represented by the Circle ABCDEBA. *Plate* 3. *Fig.* 7.

9. A *Great Circle*, is either the Primitive Circle, a Right Circle, or an Oblique Circle.

These Circles considered severally, or jointly, afford divers Problems, which are the subject Matter of *Spheric Geometry*, and are such as follow.

Problem I. *To find the Pole of any Great Circle.*

Definition 1. A *Great Circle*, is either the Primitive Circle, as BCDEB; or a Right Circle, as the Diameter BAD; or an Oblique Circle, as Arc BFD. *Plate* 3. *Fig.* 7.

2. *The Pole of a Great Circle*, is a Point every Way 90 Degrees distant from it: And

Note 1. The Pole of a Great Circle is either upon the Primitive Circle, or within it.

2. When the Pole is within, 'tis either at the Primitive Circle's Center, or not.

In this Problem are three Cases.

Problem I. Case 1. *The Pole of the Primitive Circle is required.*

Example. BCDE the Primitive Circle given; to find the Pole thereof is required. *Plate* 3. *Fig.* 7.

The Rule. Find A, the Center of the Primitive Circle BCDE, which Center A is the Pole required.

Problem I. Case 2. *The Pole of a Right Circle is required.*

Definition. A *Right Circle* passeth through the Center of the Primitive Circle, and in the Projection as a Diameter; as BAD.

Note, A Right Circle hath its Pole on the Primitive Circle.

Example. The Pole of the Right Circle BAD, is required. *Plate* 3. *Fig.* 7.

The Rule. From the Chords lay 90 Degrees on the Primitive Circle from B or D, both ways to C and E; I say, C, and E, are the Poles of the Right Circle BAD.

Problem I. Case 3. *The Pole of the Oblique Circle is required.*

Definition. An *Oblique Circle* passeth not through the Center of the Primitive Circle, and in the Projection is represented by an Arc; As BFD. *Plate* 3. *Fig.* 7.

Note,

Note 1. The Poles of an Oblique Circle, are in a Diameter which passeth through its Center.

2. One of the Poles of an Oblique Circle, lieth between the Centers of the Primitive, and Oblique Circle.

3. Every Great Circle, whether Right, or Oblique, cutteth the Primitive diametrically opposite.

Example.

BCDE the Primitive BFD the Oblique	Circle, and	A y	its Center given.

The Pole of the Oblique Circle BFD *is required. Plate* 3. *Fig.* 7.

The Rule. 1. Through A and y, draw a Diameter to cut the Primitive Circle in C and E, and the Oblique Circle in F.

2. Lay a Scale on B and F, to cut the Primitive Circle in g; which is called Reducing F, to the Primitive Circle.

3. Take 90 Degrees (from the Scale of Chords) and lay it on the Primitive Circle, from g both ways to h.

4. Reduce h to the Diameter CAE, by laying a Scale on B and h, to cut the Diameter C A E, both within the Primitive Circle, and without, either of which Points, I is the Pole of the Oblique Circle BFD.

Problem II. *To describe a Spheric Angle.*

Definition. A *Spheric Angle*, is made by the Intersection of two Great Circles; the Intersection being the Angular Point.

Note, In this Problem are two Cases.

Problem II. Case 1. *To make an Angle, that the Angular Point may be at the Center of the Primitive Circle.*

The Rule. Such an Angle is made (in all Respects) like a Plane Angle.

Example. An Angle B A C equal 40d. 30m. (whose Angular Point A may be the Center of the Primitive Circle) is required to be made. *Plate* 3. *Fig.* 8.

1. With a Chord of 60 Degrees (on the Center A) describe the Primitive Circle, BCDE.

2. On the Primitive Circle, and from the same Chords, make BC equal to 40d. 30m.

3. From B and C, draw two Right Circles, or Diameters, thro' A, which will include the Angle BAC required to be made.

Problem II. Case 2. *To make an Angle, that the Angular Point may be at the Primitive Circle.*

The Rule. Such an Angle is made by drawing an Oblique Circle, with the Secant of the given Angle.

Example

Example. *An Angle* EBF *equal to* 34d. 30m. *(whose Angular Point* B *may be at the Primitive Circle) is required to be made?* Plate 3. Fig. 7.

1. Describe the Primitive Circle BCDE, as before directed.

2. Lay a Scale on A (the Primitive Circle's Center) and cut the Primitive Circle in B and D.

3. With the Secant of the given Angle 34d. 30m. and one Foot in B, describe an Arc y.

4. With the same, and one Foot on D, cross the former Arc in y; the Center of the Oblique Circle BFD, which will include the Angle EBF eqal to EDF, required to be made.

Note; When the given Angle is Obtuse, take its Supplement to 180 Degrees, and with the Remainder make the Angle as above directed, and 'tis done.

Prob. III. *To draw a great Circle through any given Point so that it shall make at the Primitive Circle any given Angle.*

The Rule. 1. With the Tangent of the given Angle, and one Foot on the Center of the Primitive Circle, make an Arc.

2. With the Secant of the same, and one Foot in the given Point, cut the former Arc, which Point of Intersection, is the Center of the Circle required to be drawn.

Example. *Plate* 3. *Fig.* 7.

BCDE the Primitive Circle }
A the Center thereof - - } given;
F the Point - - - - - }

Through F, to draw an Oblique Circle, that it may make an Angle at the Primitive Circle, equal to 34d. 30m. is required?

Note; The given Point must be so far from the Center of the Primitive Circle, that the Tangent from the Center, and the Secant (of the same) from the given Point may intersect, or cut each other; otherwise 'tis impossible.

1. With the Tangent of 34d. 30m. and one Foot in A, make an Arc y.

2. Then with the Secant of the same, and one Foot in F, cut the Arc y, in the Center of the Oblique Circle BFD, required to be drawn; and if B and D are diametrically opposite, 'tis done true, otherwise not.

Problem IV. *To draw a Great Circle, through any two Points given; either both within the Primitive Circle, or one within, and the other without.*

The Rule. 1. Draw a Line from the Primitive Circle's Center, through one (always the remotest) of the two Points, to cut the Primitive Circle, and produce it at Pleasure.

2. From the said Point draw another Line to a Point in the Primitive Circle, that is 90d. Distance from the first Line.

3. On the last Line, and at the Point in the Primitive Circle, erect a perpendicular, to cut the first Line in the third Point.

4. Through the two given Points, and this last third Point, draw (by Chapter 1. Section 2. Problem 7. of *Practical Geometry*, in Page 15.) an Arc of a Circle, and 'tis done.

Example. *Plate* 3. *Fig.* 9.

BCDE the Primitive Circle }
A the Center thereof - - } given;
F and G the two Points - }

Through F and G 'tis required to draw a great Circle?

1. Through A and F, draw the Diameter BFAD, to cut the Primitive Circle in B and D, and continue it farther at Pleasure.

2. Lay the Chord of 90d. on the Primitive Circle from B, or from D, to C or E, and draw the Line FC or FE.

3. At C erect CH perpendicular to CF, or at E erect EH perpendicular to EF, to cut the Diameter BFAD in H, the third Point.

4. Through F, G, and H, draw a Circle IFGKH, which will cut the Primitive Circle in I and K, diametrically opposite, and 'tis done.

Problem V. *To draw a great Circle perpendicular to, or at Right Angles, with a given Great Circle.*

A General Rule. Draw a Great Circle to pass through the Poles of the given Great Circle, and 'tis perpendicular to it, or it makes a Right Angle with it.

Note, In this Problem are four Cases.

Problem V. Case 1. *To draw a great Circle perpendicular to the Primitive Circle.*

The Rule. This is done by drawing a Diameter through the Center of the Primitive Circle; for the Center of the Primitive Circle being its Pole, all Right Lines drawn through the Center, represent perpendicular Circles to the primitive Circle.

Example. *Plate* 3. *Plate* 7.

BCDE the Primitive Circle, and A its Center given; to draw a Great Circle perpendicular to it, is required?

Through A draw the Right Circle BAD, and it is perpendicular to the Primitive one BCDE, as was required.

Problem

Problem V. Case 2. *To draw a Right Circle perpendicular to a given Right Circle.*

The Rule. This is done by drawing a Diameter at Right Angles to the given Right Circle; or quartering the Primitive Circle (by Chapter 1. Section 2. Problem 8. of *Practical Geometry*, in Page 15.) with two Diameters.

Example. *Plate* 3. *Fig.* 7.

BCDE the Primitive } Circle, { and A its Center } given;
BAD is a Right - - } Circle, { - - - - - - - } given;

To draw a Right Circle perpendicular to the Right Circle BAD, is required?

Draw the Diameter CAE perpendicular to BAD, or from the Chords lay 90d. on the Primitive Circle from B, or from D, to C and E, which are its two Poles, and through C, A, and E, draw a Diameter, and 'tis done.

Problem V. Case 3. *To draw an Oblique Circle perpendicular to a given Right Circle.*

The Rule. 1. Find the two Poles of the given Right Circle, by Problem 1. Case 2. of *Spheric Geometry*, in Page 111.

2. Draw an Oblique Circle through those two Poles, and it is done.

Example. *Plate* 3. *Fig.* 7.

BCDE the Primitive Circle }
A the Center thereof - - } given;
BAD is a Right Circle - - }

To draw an Oblique Circle, perpendicular to the Right Circle BAD, is required?

1. Take the Chord of 90d. and lay it from B, or D, both ways to C, and E, which are the two Poles of the Right Circle BAD.

2. With any Distance, and one Foot on C, and E, draw Arcs, to cut each other in x, the Center of the Oblique Circle CGE, required to be drawn.

Note, If AG be known, or limited to a certain Distance, then it is done by drawing a Circle through the three Points C, G, and E.

Or, 2. If AG be any known Number of Degrees, then take the Secant of its Complement, and setting one Foot on C, or E, the other will cross the given Right Circle, in the Center of the Oblique Circle required to be drawn.

Problem V. Case 4. *To draw an Oblique Circle, perpendicular to a given Oblique Circle.*

The Rule. 1. Find the two Poles of the given Oblique Circle, by Problem 1. Case 3. of *Spheric Geometry*, in Pages 111 and 112.

2. Through those Poles draw a Great Circle, which will cut the Oblique Circle at Right Angles, and the Primitive Circle diametrically opposite, and 'tis done.

Example. *Plate* 3. *Fig.* 10.

BCDE the Primitive } Circle, and { A } its Center given;
BFD an Oblique } Circle, and { y } its Center given;

To draw another Oblique Circle perpendicular to the Oblique Circle BFD, is required?

1. Find G, the Interior Pole of the given Oblique Circle BFD, by Problem 1. Case 3. of *Spheric Geometry*, in Pages 111 and 112.

2. Through G, draw the Circle HIGK, to cut the Primitive in H and K, and the Oblique Circle BFD in some Point I, (so that HAK is in a Diameter) and 'tis done, for then the Angle BIH is a right Angle, and the Oblique Circles BFID, and HIGK, are perpendicular to one another.

Note; 1. If the Point I (on the given Oblique Circle) is given; then draw a Circle through I, (and the Interior Pole G, by Problem 4. of *Spheric Geometry*, in Page 113) and 'tis done.

Or, 2. If it be required, that the said Oblique Circle shall make a certain Angle at the Primitive One, then draw a Circle through the said Pole G, by Problem 3. of *Spheric Geometry*, in Page 113.

Problem VI. *To lay any Quantity of Degrees on any Great Circle.*

In this Problem are three Cases.

Problem VI. Case 1. *To lay any Quantity of Degrees on the Primitive Circle.*

The Rule. This is done by, or from the Scale of *Chords*.

Example. *Plate* 3. *Fig.* 8.

BCDE the Primitive Circle, and A its Center given;

To lay 40d. 30m. on the Primitive Circle from B is required?

From the Scale of *Chords*, take 40d. 30m. and lay it on the Primitive Circle from B to C, and 'tis done.

Problem VI. Case 2. *To lay any Quantity of Degrees on a Right Circle.*

The Rule. This is done from the Scale of Half-Tangents, counting the Beginning thereof to be the Center of the Primitive Circle.

Example

Example. *Plate* 3. *Fig.* 8.

BCDE the Primitive Circle, and A its Center } given;
BAD is a Right Circle - - - - - - - - }

On the Right Circle BAD, from A, to lay 40d. 30m. or from B to lay 49d. 30m. is required?

From the Half-Tangents, take 40d. 30m. and lay it on the Right Circle, BAD from A to *l*, or 49d. 3cm. the contrary Way of the Half-Tangents, laid from B to *l*, and 'tis done: For A *l* and *l* B together are equal to 90 Degrees.

Problem VI. Case 3. *To lay any Quantity of Degrees on an Oblique Circle.*

The Rule. 1. Find the Poles of the given Oblique Circle, by Problem 1. Case 3. of *Spheric Geometry*, in Pages 111 and 112.

2. Lay the given Quantity of Degrees on the Primitive Circle, by Case 1. of this Problem.

3. Reduce it from the Primitive Circle to the given Oblique Circle, by laying a Scale on either of its Poles, and 'tis done.

Example. *Plate* 3. *Fig.* 11.

BCDE the Primitive } Circle, and { A } its Center given;
BIFD an Oblique - } { y }

On the Oblique Circle BFD (from B) to lay 51d. 30m. is required?

1. By A and y, draw the Diameter CAE, and find G the Pole of the Oblique Circle, by Problem 1. Case 3. in Page 112.

2. From the Scale of *Chords*, take 51d. 30m. and lay it on the Primitive Circle, from B to H.

3. Lay a Scale on G and H, to cut the Oblique Circle, BFD in I, then is BI (on the Oblique Circle) equal to 51d. 30m. as was required.

Problem VII. *To measure any Part of a Great Circle.*

In this Problem are three Cases, which are but the converse of those in the last Problem.

Problem VII. Case 1. *To measure any Part of the Primitive Circle.*

The Rule. Take the Part required to be measured, and lay it on the Scale of Chords, and it sheweth how much it is.

Example. *Plate* 3. *Fig.* 8.

BCDE the Primitive Circle, and A its Center given;

To measure BC, a Part of the Primitive Circle, is required?

Take the Extent BC in the Compasses, and lay it on the Chords, which will shew how many Degrees doth measure BC.

Problem VII, Case 2. *To measure any Part of a Right Circle.*

The Rule. 1. If the Part to be measured lieth next the Center of the Primitive Circle, then it is measured on the Scale of Half-Tangents, from the Brass Center Pin at the beginning thereof.

2. When the Part to be measured lieth next to the Primitive Circle, then it is measured on the Scale of Half-Tangents, from 90d. counting 80d. to be 10d. 70 to be 20, 60 to be 30, &c.

Example. *Plate* 3. *Fig.* 8.

BCDE the Primitive Circle, and A its Center } given;
BAD a Right Circle - - - - - - - - }
To measure *A l*, or *B l*, on the Right Circle *BAD*, is required?

1. Take *A l*, and lay it on the Scale of Half-Tangents, from the Brass Center Pin (at the Beginning of it) which will shew how many Degrees it is: Or,

2. Take *B l*, and lay it on the Scale of Half-Tangents, from 90d. backwards, counting 80d. to be 10, and 70 to be 20, &c. which will shew how many degrees *B l* is: And,

Note, That *A l*, and *B l*, will make together just 90d. they being Complements to each other.

Prob. VII. Case 3. *To measure any Part of an Oblique Circle.*

The Rule. 1. Find the Poles of the given Oblique Circle, by Problem 1. Case 3, of *Spheric Geometry*, in Pages 111 and 112.

2. Lay a Scale on either of the said Poles, and the Part desired to be measured, and reduce it to the Primitive Circle.

3. Being thus reduced to the Primitive Circle, 'tis measured on the Scale of Chords, as before in Case 1. of this Problem.

Example. *Plate* 3. *Fig.* 11.

BCDE the Primitive } Circle, and { A } its Center given;
BIFD an Oblique } { y }
To measure BI, and FI, on the Oblique Circle BIFD is required?

4. Find G the Pole of the Oblique ircle BIFD, by Problem 1. Case 3. of *Spheric Geometry*, in Pages 111 and 111.

2. Lay a Scale on G and I, to cut the Primitive Circle in H,

3. Then

3. Then EH measured on the Scale of Chords is the Measure of FI; and BH on the same Scale of Chords is the Measure of BI.

Problem VIII. *To measure any Spheric Angle.*

In this Problem are four Cases; and this

A General Rule. A Spheric Angle is measured by the Arc of a Great Circle, intercepted between the two containing Sides, the Angular Point being the Pole of that Circle: Or the Distance of the Poles of the containing Sides, as equal to the Measure of the contained Angle.

Prob. VIII. Case 1. *To measure an Angle, when its Angular Point is the Center of the Primitive Circle.*

The Rule. Such an Angle is measured (like a Plane Angle) on the Primitive Circle, by a Scale of Chords.

Example. *Plate* 3. *Fig.* 8.

BCDE the Primitive Circle, A its Center, and the Angular Point given; to measure the Angle BAC is required?

Take BC, and measure it on the Scale of Chords, shews the Angle BAC, how much it is.

Prob. VIII. Case 2. *To measure an Angle, when its Angular Point is at the Primitive Circle.*

The Rule. 1. Find the Poles of the two containing Sides, by Problem 1. of *Spheric Geometry*, in Pages 111 and 112.

2. The Distance of these Poles, is the Measure of the required Angle.

Note 1. When the two Poles are in one Diameter, or Right Circle, it is measured on the Scale of Half-Tangents.

2. When they are not in one Diameter, then reduce them to the Primitive Circle, by laying a Scale on the Angular Point, and the said two Poles, which Distance being measured, on the Scale of Chords, is the Measure of the required Angle.

Example. *Plate* 3. *Fig.* 8.

BCDE the Primitive } Circle, and { A } its Center given.
BGFD an Oblique } { y }

To measure the Angle EDF, *equal to the Angle* EBF *is required.*

1. Through A and y, draw a Diameter to cut the Primitive Circle in H and I, and the Oblique Circle in G.

2. In the Diameter IAH, find K, the Pole of the Oblique Circle BGFD.

3. The Diſtance AK, or GI, meaſured on the Scale of Half-Tangents, (the latter the contrary Way on that Scale from 90d.) ſheweth how much the Angle EDF, or EBF is.

Problem VIII. Caſe 3. *To meaſure an Angle when its Angular Point is not the Center of, nor at the Primitive Circle.*

The Rule is this; 1. Find the two Interior Poles of the two containing Sides, by Problem 1. in Pages 111 and 112.

2. Reduce thoſe two Poles to the Primitive Circle, then meaſure the Diſtance of them on a Scale of Chords, and 'tis done.

Note, Reduce, is to lay a Scale on the Angular Point (required to be meaſured) and the ſaid two Poles, to cut the Primitive Circle.

Example. *Plate* 3. *Fig.* 8.

BCDE *the Primitive*	} *Circle, and*	{ A	} *its Center* - -	} given.
BGFD *an Oblique*		y		
CAFE *a Right*			*cuts the Oblique in* F.	

To meaſure the Angle DFE, *or* BFE *is required?*

1, Through A and y, draw the Diameter IAH, and in it find K the Interior Pole of the Oblique Circle BGFD, as before in the laſt Caſe.

2. Find L the Pole of the Right Circle CAFE, by Problem 1. Caſe 2. of *Spheric Geometry*, in Page 111.

3. Reduce K to the Primitive Circle (by laying a Scale on F and K, to cut the Primitive Circle) and 'tis M; then L M meaſured on the Chords, ſheweth how much the Angle DFE, or BFE, is; one Acute, and the other Obtuſe, being the Supplement of the former.

Problem VIII. Caſe 4. *To meaſure an Angle when the containing Sides are both Oblique Circles.*

Example. *Plate* 3. *Fig.* 12.

BCDE *the Primitive*	} *Circle, and*	{ A	} *its Center given;*
CHFE *an Oblique*		y	
BGFD *an Oblique*		x	

To meaſure the Angle DFE, *or* BFE *is required?*

1. By A and y; and A and x, draw two Diameters; in them find I and K, the two Interior Poles of the containing Sides, by Problem 1. Caſe 3. of *Spheric Geometry*, in Pages 111 and 112.

2. Reduce thoſe two Poles I and K, (by laying a Scale on the Angular Point F, and them) to cut the Primitive Circle in L and M; which being meaſured on the Chords, ſheweth how much the Angle DFE, or BFE, is; one Acute, the other Obtuſe.

Problem

Problem IX. *To draw a Parallel Circle.*

Definition. A lesser Circle of the Sphere, cutteth it into two unequal Parts, and when parallel to a given Great Circle, is called a Parallel Circle.

In this Problem are three Cases,

Problem IX. Case 1. *To draw a Circle parallel to the Primitive Circle, at any given Distance from it, or from its Pole.*

The Rule. With the Half-Tangent of its Distance from the Pole, and one Foot on the Center of the Primitive Circle, draw a Circle, and it is done.

Example. *Plate* 3. *Fig.* 7.

BCDE the Primitive Circle, A its Center given; to draw a Circle parallel to BCDE, at 30d. Distance from it is required?

With the Half-Tangent of 60d. (the Complement of 30d. and) its Distance from the Pole of the Primitive Circle, set one Foot in A (the Center, and pole of the Primitive Circle) and describe the Circle *t m n o*, which is parallel as required.

Problem IX. Case 2. *To draw a Circle parallel to a Right Circle.*

The Rule. 1. From the Chords lay the Parallel's Distance from the Right Circle, or the Complement thereof, from one of the Poles of the Right Circle, both Ways, and note these two Marks on the Primitive Circle.

2. With the Tangent of the Parallel's Distance from the Pole of the Right Circle, and one Foot on each of those two Marks, describe Acres to cut each other, in the Center of the Parallel Circle required to be drawn.

Example. *Plate* 3. *Fig.* 7.

BCDE the Primitive Circle, and A its Center } given;
BAD is a Right Circle - - - - - - - - }

To draw a Circle parallel to BAD, *at* 40d. *distant from it, or* 50d. *Distance from* C, *its Pole is required?*

1. Lay 40d. from B to *p*, and from D to *q*; or lay its Complement 50d. from C, one of the Poles of the Right Circle both Ways to the said *p* and *q*.

With the Tangent of 50d. and one Foot on { *p* / *q* } make the Arc *y*, which Point of Intersection at *y*, is the Center of the parallel Circle *p r q*, required to be drawn.

Problem

Problem IX. Case 3. *To draw a Circle parallel to an Oblique Circle.*

The Rule. 1. Find the interior Pole of the given Oblique Circle by Problem 1. Case 3. in Pages 111 and 112, which reduce to the Primitive Circle, and therefrom lay the Parallels Distance from the Pole both Ways, which being reduced to the Right Circle, is the Diameter of the parallel one.

2. Find the middle of the said Diameter, which is the Center of the parallel Circle passing through those Marks, and it is done.

Example. *Plate* 3. *Fig.* 11.

BCDE the Primitive } Circle, and { A } its Center given.
BFD an Oblique } Circle, and { y } its Center given.

To draw a Circle parallel to BFD, at 40d. Distance from it, or 50d. Distance from its Pole is required?

1. Find G the Pole of the given Oblique Circle B F D, by Problem 1. Case 3. of *Spheric Geometry*, in Pages 111 and 112.

2. Measure AG on the Half-Tangents, and suppose it to be 30d. which add to, and subtract from 50; the Sum 80d. lay from A to *k*; and the Difference 20d. lay from A to *c*: Or thus; reduce G to the Primitive Circle, and from it lay 50d. both Ways, which reduce to the Right Circle, gives the same Point *k* and *c*.

3. Find *m* the middle between *k* and *c*, and on *m* as a Center, draw a Circle to pass through *k* and *c*, which is the parallel Circle required to be drawn.

Thus much for Problems necessary for making and measuring *Spheric Triangles*; which I advise the Learner perfectly to acquaint himself with, and then the following Applications will be the better understood.

Section II: *The Use of the Nine preceding Problems, in making any* Spheric Triangle, *and measuring its Parts, or the Doctrine of* Spheric Triangles.

Definition 1. A Sphere or Globe, is a round Body made by the moving a Semi-circle about its own Diameter, 'till the Motion ends where it first began: The Semi-circle's Diameter is the Axis or Diameter of the Sphere; in the Middle of which is a Point called the Center, from whence all Right Lines drawn to the Surface, or Outside of the Sphere, are equal.

2. A *Spheric Triangle*, is described on the Surface of the Sphere, whose Sides are the Arcs of three Great Circles mutually

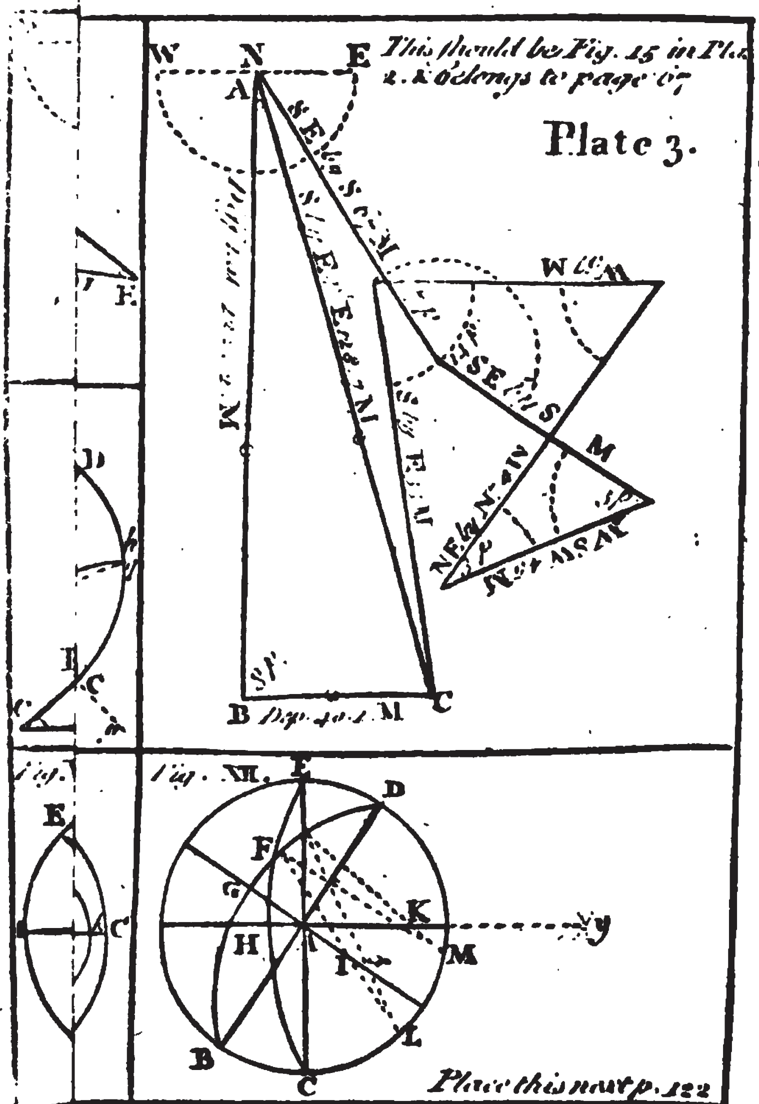
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Plate 3.
W
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Fig. XII.
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ally interfecting each other; and is either Quadrantal, Rectangular, or Obliquangular.

3.	A Quadrantal A Rectangular An Obliquangular	Triangle hath	one Side one Angle	90d.
			no Side a Quadrant,	

or 90d. nor any Angle 90d.

Note 1. In every Spheric Triangle, each Side is lefs than a Semi-circle, or 180 Degrees.

2. The Sum of any two Sides is greater than the third Side.

3. The three Sides added together, their fum is lefs than 360 Degrees.

4. The Sum of the three Angles is ever more than two Right Angles, or 180d. but always lefs than fix Right, or 540 Degrees.

Problem X. *The Hypothenufe, and one Leg given; to make a Rectangular Spheric Triangle.*

Note, To make a Rectangle Spheric Triangle, two Things, (befides the Rectangle) muft be given.

Example. *Plate* 4. *Fig.* 1.

The { Hypothenufe AC 54d. 25m. / Leg BC - - - 23d. 29m. } given;

With them to make a Rectangular Spheric Triangle is required?

1. Draw the Primitive Circle with the Chord of 60d. or Sine of 90d. or with a Half-Tangent of 90d. and quarter it, which muft be done in every Problem.

Then confider whether one of the Oblique Angles fhall be at the Primitive Circle, or at its Center.

Firft, If at the Center of the Primitive Circle, then

1. Draw (by Problem 9. Cafe 1. of *Spheric Geometry*) a Circle parallel to the Primitive, at 54d. 25m. (the given Hypothenufe) Diftant from its Pole; that is, with the Half-Tangent of 54d. 25m. and one Foot on A (the Center of the Primitive Circle) defcribe a Circle.

2. Then (by the fecond Cafe of the aforefaid Problem) draw a Circle parallel to a Right Circle, at 23d. 29m. the (given Leg) diftant from it; thefe two parallel Circles cut each other in C.

3. Through A and C, draw a great Circle, which in this Cafe is a Right Circle) that is a Diameter; and from C (by Problem 5. Cafe 1. of *Spheric Geometry*, in Page 114) draw a Great

Great Circle, perpendicular, or at Right Angles to A B, which in this Case will be an Oblique Circle, and 'tis done.

Or,

Secondly, To make the Triangle that one of its Oblique Angles may be at the Primitive Circle.

1. The Circle being described, and quartered as before directed; Then,

2. Draw (by Problem 9. of *Spheric Geometry*) a Circle parallel to a Right Circle, at 54d. 25m. distant from A its Pole.

3. And by the aforesaid Problem draw a Circle parallel to the Primitive Circle, at 23d. 29m. distant from it, that is, take 66d. 31m from the Scale of Half-Tangents, and with one Foot on the Center draw a Circle, to cut the former Parallel Circle in C.

4. Then through A and C draw a Great Circle, which in this Case will be an Oblique Circle; and from C by (Problem 5. Case 1. of *Spheric Geometry*, in Page 114) draw a Great Circle perpendicular to AB; which in this Case will be a Right Circle and 'tis done.

To measure the Things required.

The { Leg AB - - - - - is } measured } 7. Page 117.
{ Angles BAC and ACB are } by Prob. } 8. Page 118, 119.

Problem XI. *The Hypothenuse and one Angle given, to make a Triangle.*

Example. *Plate* 4. *Fig.* 2.

The { Hypothenuse AC 54d. 25m. } given;
{ Angle BAC - 29d. 30m. }

With them to make a Rectangular Sphere Triangle, is required?

First, To make the Triangle, that the given Angle BAC may be at the Center of the Primitive Circle.

1. Having described that Circle, and quarter'd it, as before directed in Problem 10. make the Angle BAC equal to 29d. 30m. (by Problem 2. Case 1. in Page 112. of *Spheric Geometry*) that is, from the Scale of Chords, lay 29d. 30m. on the Primitive Circle, and therefrom draw a Line through A, the Center of the Primitive Circle.

2. Make AC equal 54d. 25m. (by Problem 6. Case 2 in Page 117. of *Spheric Geometry*) that is, from the Scale of Half-Tangents, lay 54d. 25m. from A to C.

By Problem 5. Case 3. in Page 115. draw a Great Circle thro'

thro' C, perpendicular unto A B, to cut the Right Circle A B in B; which in this Case is an Oblique Circle; and 'tis done.

Secondly, To make the Triangle so, that the given Angle may be at the Primitive Circle.

1. After the Primitive Circle is made, and quartered as before let A be one of those Marks or Quarters; then (by Problem 2. Case 2. in Page 112.) make the Angle at A equal to 29d. 30m. by drawing an Oblique Circle with the Secant of the given Angle, as is the Oblique Circle AC.

2. Then (by Problem 6. Case 3. in Page 117.) lay 54d. 25m. on the Oblique Circle from A to C; Or draw (by Problem 9. Case 2. in Page 121.) a Circle parallel to a Right Circle at 54d. 25m. distant from A its Pole, to cut the Oblique Circle AC in C.

3. By Problem 5. Case 1. in Page 114, draw a Circle from C, perpendicular to the Primitive Circle AB, to cut AB, in B; which in this Case is a Right Circle, and 'tis done.

To measure the Things required.

The	Legs AB, and BC are	measured by Prob.	7.	Page 118.
	Angle ACB - - is		8.	Page 119.

Problem XII. *A Leg, and its adjacent Angle given; to make a Triangle.* Example. *Plate* 4. *Fig.* 3.

The	Leg AB - -	50d. 30m.	given;
	Angle BAC -	29d. 30m.	

With them to make a Rectangle Spheric Triangle is required;

First, To make the Triangle, that the given Angle (BAC) may be at the Primitive Circle's Center.

1. The Primitive Circle being described, and quartered as before, and A placed at its Center; (then by Problem 2. Case 1. in Page 112. of *Spheric Geometry*) make the Angle (BAC) equal 29d. 30m. by drawing the Right Circle AC.

2. By Problem 6. Case 2. in Page 112. make AB equal to 50d. 30m. that is, from the Scale of Half-Tangents, lay 50d. 30m. on the Right Circle from A to B.

3. By Problem 5. Case 3. in Page 115. draw through B, an Oblique Circle BC, perpendicular to the Right Circle AB, to cut the Right Circle AC in C, and its done. Or thus,

From the Scale of Secants, take 39d. 30m. the Complement of AB 50d. 30m. setting one Foot in B, the other Foot marks out (on the Right Circle AB extended) the Center of the Oblique Circle BC, which concludes the Triangle.

Secondly, To make the Triangle so, that the given Angle may be at the Primitive Circle.

1. A

1. A being on, or at the Primitive Circle, (then by Problem 2. Case 2. in Page 113.) made the Angle BAC equal to 29d. 30m. by drawing the Oblique Circle AC, with the Secant thereof.

2. By Problem 6. Case 1. in Page 114. make AB (on the Primitive Circle) equal to 50d. 30m. by laying the Chord of it from A to B.

3. By Problem 5. Case 1. in Page 114 draw a Great Circle from B, perpendiclar to AB, as the Right Circle BC, to cut the Oblique Circle AC in C, and 'tisdone.

To measure the Things required.

The { Leg BC, and Hypot. AC are / Angle ACB - - - is } measured by Prob { 7. / 8. }

Problem XIII. *A Leg and its opposite Angle given; to make a Triangle.*

Example. *Plate* 4. *Fig.* 2.

The { Leg BC - - 23d. 29m. / Angle BAC 29d. 00m. } given;

With them to make a Rectangular Spheric Triangle is required?

First, To make the Triangle, with the given Angle at the Primitive Circle's Center.

1. The Primitive Circle drawn, &c. as before, and A at its Center, (by Problem 2. Case 1. in Page 112.) make the Angle BAC equal to 29d. 30m. by drawing two Right Circles AB, and AC.

2. Draw (by Problem 9. Case 2. in Page 121.) a Parallel Citcle, parallel to the Right Circle AB, at 23d. 29m. distance from it, to cut AC in C.

3. From C (by Problem 5. Case 3. in Page 115.) draw an Oblique Circle BC, perpendicular to the Right Circle AB to cut AB in B, and 'tis done.

Secondly, To make the Triangle, that the given Angle may be at the Primitive Circle.

1. A, being at the Primitive Circle, by Problem 2. Case 2. in Page 113.) make the Angle BAC equal to 29d. 30m. by drawing the Oblique Circle AC, with the Secant thereof.

2. Py Problem 9. Case 1. in Page 121. draw a lesser Circle, parallel to the Primitive Circle AB at 23d. 29m. distance from it, to cut the Oblique one AC in C: Or thus, with the Half-Tangent of 66d. 31m. (the Complement of 23d. 29m.) and one Foot in the Center of the Primitive Circle, with the other cut the Oblique Circle in C.

3. From C (by Problem 5. Case 1. in Page 114. draw a Right Circle

Circle BC, perpendicular to the Primitive Circle AB, to cut AB in B, and 'tis done.

To measure the Things required.

The { Hypot AC, and Leg AB, are / Angle ACD - - - is } measured by Prob. { 7. / 8. } in Pages 117, 118 and 119.

Problem XV. *Both Legs given; to make a Spheric Triangle.*

The Leg { AB 50d. 30m. / BC 23d. 29m. } given;

With them to make a Rectangular Spheric Triangle is required?

First, To make the Triangle, that one of the Oblique Angles may be at the Primitive Circle's Center.

1. A, being at the Center of the Primitive Circle, draw the Right Circle AB, and thereon by Problem 6. Case 2. in Page 117) make AB equal to 50d. 30m. by laying the Half-Tangent thereof from A to B.

2. From B (by Problem 5. Case 3. in Page 115.) draw an Oblique Circle BC perpendicular to AB; Or thus,

With the Secant of 39d. 30m. (the Complement of 50d. 30m.) and one Foot on B, with the other Foot mark out (in the Right Circle AB extended) the Center of the Oblique Circle BC.

3. Draw (by Problem 9. Case 2. in Page 121.) a lesser Circle, parallel to the Right Circle AB, at 23d. 29m. distance from it, to cut the Oblique Circle BC in C: Or thus,

By Problem 6. Case 3. in Page 117. lay 23d. 29m. on the Oblique Circle, from B to C.

4. Through A and C draw a Great Circle; which in this Case is a Right one, and 'tis done.

Secondly, To make the Triangle, that one of the Oblique Angles may be at the Primitive Circle.

1. By Problem 6. Case 1. in Page 116. make AB equal 50d. 30m. by laying the Chord thereof on the Primitive Circle from A to B.

2. From B (by Problem 5. Case 1. in Page 114. draw a Great Circle perpendicular to the Primitive Circle, (which in this Case is a Right Circle) as BC.

3. Draw by Problem 9. Case 1. in Page 121.) a Lesser Circle, parallel to the Primitive Circle AB, 23d. 29m. distant from it, to cut the Right Circle BC in C: Or thus.

From the Scale of Half-Tangents, lay 66d. 31m. (the Complement of 23d. 29m.) from the Center of the Primitive Circle, on the Right Circle to C.

4. Through

4. Through A and C, draw a Great Circle; which in this Case is an Oblique one, and 'tis done.

To measure the Things required.

The { Hypothenuse AC - - is / Angles BAC, and ACB are } measured by Problem { 7. / 8. } in Pages 117, 118, and 119.

Problem XV. *Both the Angles given; to make a Spheric Triangle.*

Example. *Plate* 4. *Fig.* 6.

The Angle { BAC 29d. 30m. / ACB 71d. 56m. } given;

With them to make a Rectangle Spheric Triangle is required;

1. By Problem 2. Case 2. in Page 113, make the Angle BAC equal to 29d. 30m. by drawing the Oblique Circle AB, with the Secant thereof,

2. Find I, the Pole of the Oblique Circle AB. by Problem 1. Case 3. in Pages 111 and 112.

3. Through I (by Problem 3. in Page 113.) draw an Oblique Circle IBC, that the Angle ACB may be 71d. 56m. which will cut the Oblique Circle AB in B the Rectangle, and the Primitive Circle in C, and so complete the Rectangle Spheric Triangle ABC, required to be made.

To measure the Things required,

The Hypothenuse AC, and Legs AB, and BC, are measured by Problem 7. of *Spheric Geometry*, in Pages 118 and 119.

Thus, All the Cases of Rectangular Spheric Triangles may be described, and resolved by the last Six Problems: In like manner, the Cases of Oblique Spheric Triangles may be Geometrically solved by the Six following.

Problem XVI. *Two Sides and an Angle opposite given; to make an Oblique Spheric Triangle.*

Note; To make an Oblique Spheric Triangle, three Things must be given.

Note also, The Angle opposite to the other given Side ought to be foreknown, whether Acute or Obtuse; otherwise two several Triangles may be made from the same given Things.

Example. *Plate* 4. *Fig.* 7.

The { Side - - AC 34d. 07m. / Side - - AD 65d. 20m. / Angle ADC 27d. 30m. } given;

With them to make a Spheric Triangle (so that the Angle ACD may be Obtuse) is required?

1. On

1. On the Primitive Circle, make AD (the given Side joining to the given Angle) equal to 65d. 20m. by Problem 6. Case 1. of Spheric Geometry, in Page 116.

2. By Problem 2. Case 2. in Page 112. make the Angle ADC equal to 27d. 30m. by drawing the Oblique Circle DC, with the Secant thereof.

3. Draw (by Problem 9. Case 2. in Page 121) a Parallel Circle at 34d. 07m. distance from A, to cut the Oblique Circle DC in C.

4. Through A and C, draw a Great Circle and 'tis done.

To measure the Things required.

The { Angles CAD, and ACD are / Side CD — — is } measured by Prob. { 8. / 7. } in Pages 118 and 119.

Problem 17. *Two Angles, and one Side opposite given; to make an Oblique Spheric Triangle.*

Example. *Plate* 4. *Fig.* 7.

The { Angle CAD 30d. 48m. / Angle ADC 27d. 30m. / Side CD - 38d. 28m. } given;

With them to make an Oblique Spheric Triangle, so that the Side AC may be less than a Quadrant, or 90d. is required?

1. By Prob. 2. Case 2. in Page 112, make the Angle ADC equal to 27d. 30m. (always the Angle joining to the given Side) by drawing the Oblique Circle DC, with the Secant of it.

2. Draw (by Prob. 9. Case 2. in Page 121) a parallel Circle at 38d. 28m. distance from D, to cut the Oblique Circle DC, in C.

3. Then through C (by Prob. 3. in Page 113.) draw a great Circle to make an Angle at the Primitive Circle that may be equal to 30d. 48m. as is the Oblique Circle CA, and 'tis done.

To measure the Things required.

The { Side AC, and AD are / Angle ACD —— is } measured by Prob. { 7. / 8. } in Pages 118 and 119.

Note; In these two last Problems, the three given Things without the Quality of a Fourth, are not sufficient to determine the Triangle; And the Quality of a Fourth is not always discoverable by the given Things; therefore they are called *Doubtful Cases.*

Problem 18. *Two Sides and one Angle between them given; to make an Oblique Spheric Triangle.*

Example. *Plate* 4. *Fig.* 9.

The { Side - - - AC 34d. 06m. / Side - - - AD 65d. 20m. / Angle - - CAD 30d. 46m. } given;

With them to make an Oblique Spheric Triangle is required?

1. By Problem 2. Case 2. in Page 112. make the Angle CAD equal to 30d. 46m. by drawing the Oblique Circle AD with the Secant thereof.

2. Make AD (on the Primitive Circle) equal to 34d. 06m. and AD (on the Oblique Circle) equal to 65d. 20m. by Problem 6. Cases 1 and 3. in Pages 116 and 117.

3. Through C and D, draw a Great Circle, and 'tis done.

To measure the Things required.

The { Side - - CD - - is / Angles ACD, and ADC are } measured by Prob. { 7. / 8. } in Pages 118 and 119.

Problem 19. *Two Angles, and one Side between them given; to make an Oblique Spheric Triangle.*

Example. *Plate* 4. *Fig.* 10.

The { Angle ACD 131d. 34m. / Angle ADC 27d. 30m. / Side - CD 38d. 28m. } given;

With them to make an Oblique Spheric Triangle is required?

1. On the Primitive Circle, make CD equal 38d. 28m. the given Side, by Problem 6. Case 1. in Page 116.

2. At D by Prob. 2. Case 2. in Pages 112 and 113) make the Angle ADC equal to 27d. 30m. by drawing the Oblique Circle DA; and at C, make the Angle ACD equal to 131d. 34m. by drawing the Oblique Circle AC, and 'tis done.

To measure the Things required.

The { Sides AC, and AD are / Angle CAD - - - } measured by Prob. { 7. / 8. } in Pages 118 and 119.

Prob. 20. *Three Sides given; to make an Oblique Spheric Triangle.*

Example. *Plate* 4. *Fig.* 11.

The Side { AD 65d. 20m. / CD 38d. 26m. / AC 34d. 08m. } given;

With them to make an Oblique Spheric Triangle is required?

1. On the Primitive Circle make AD equal to 65d. 20m. (the greater given Side) by Problem 6. Case 2. in Page 116.

2. Draw (by Prob. 9. Case 2. in Page 121.) a Parallel Circle at 34d. 8m. distance from A; and likewise another at 38d. 26m. distance from D, to cut each other in C.

3. Through

3. Through A and C, and also D and C, draw Great Circles, which will form the Spheric Triangle required.

To measure the Things required.

1. Find (by Problem 1, in Pages 111 and 112.) the Pole of the three Sides of the Triangle ACD; that is, E the Poles of AD, F the Pole of AC, and G the Pole of CD.

2. Through F and G, draw a Great Circle, of which the angular Point C is it's Pole, and you will form the Triangle EFG.

3. The Sides of the Triangle EFG, are equal to the Angles of the Triangle ACD; and the Angles of the first, are equal to the Sides of the latter, only the greater Angle in the one, is equal to the Supplement (of the greater Side in the other) to a Simi-circle.

That is; the Side EF is equal to the Angle CAD, the Side EG is equal to the Angle ADC, and the Side FG is equal to the Supplement to 180d. of the Angle ACD.

4. The Sides EF, EG, and FG, are measured by Problem 7, in Pages 117 and 118.

Prob. 21. *Three Angles given; to make an Oblique Spheric Triangle.*

Example. *Plate* 4. *Fig.* 12.

The Angle { CAD 131d. 34m. / ACD 30d. 45m. / ADC 27d. 30m. } given;

With them to make an Oblique Spheric Triangle, is required?

1. Suppose a Triangle EFG, whose Sides are equal to the Angles of the Triangle A D C; that is, the Side EF equal to 48d. 26m. the Supplement to 180d. of the Angle A C D equal to 131d. 34m. and the Side EG equal to the Angle A C D 30d. 45m. and the Side FG equal to the Angle ADC 27d. 30m.

2. Then construct the Triangle EFG with the Sides EF, EG and FG, as before in the last Problem.

3. Find (by Problem 1.) the Poles of the three Sides of the Triangle EFG; that is, A the Pole of EF, C the Pole of EG, and D the Pole of FG.

4. Through C and D, draw a Great Circle, of which the angular Point G is its Pole, which will form the Triangle ACD required to be made.

To measure the Things required.

The Sides AC, AD, and CD, are measured by Problem 7, in Pages 117 and 118.

Thus much I thought necessary as preparative to Spheric Trigonometry by Calculation, that the Learner might know how to make or construct, and measure the Spheric Triangle, or any of its Parts; a thing not well understood by many who pretend to be Teachers, to the great Disadvantage of the ingenious Scholar.

Section III. *Of the* Affections, *or* Natural Properties *of Spheric Triangles.*

THese *Affections* are carefully to be minded, in order to the understanding of what may be given, or required in a Spheric Triangle; and having already given you the several Definitions of a Sphere, and the Circles of it, with the Kinds of Triangles relative thereto, which I will not here repeat, I pass on to the Affections of Spheric Triangles, as follow.

1. Every Side of a Spheric Triangle is an Arc of a Great Circle, and is less than a Semi-circle.

2. Every Great Circle divides the Sphere or Globe into two equal Parts called Hemispheres.

3. Any two Great Circles must cut each other, and the Angles which are opposite and contrary, are equal to one another.

4. The Sum of any two sides (of a Triangle) is greater than the third Side.

5. The Sum of the three { Sides / Angles } is less than { 390d. / 540d. but more than 180d. }

6. The Sum of any two Sides, is less than the Difference between the third Side and 180d. But,

The Sum of any two Angles is more than the Difference between the third Angle and 180d.

7. In a Rectangular Triangle, the Legs and their opposite Angles are of the same Affection; that is, if a Leg be more, or less than a Quadrant, its opposite Angle is likewise more, or less, than a Right Angle.

8. In a Rectangular Triangle, if one Leg be a Quadrant, the Hypothenuse will be a Quadrant; but if both Legs be of the same Affection, the Hypothenuse is less than a Quadrant; if of different, 'tis more; and on the contrary, if each of the Legs be less, or more than 90d. the Hypothenuse is less than 90d. But, if one Leg be more than 90d. and the other less, the Hypothenuse is more than 90d.

9. If the Hypothenuse be { less / more } than 90d, one Leg, and its opposite Angle will be { less than 90d. the other may, or not. / more, and the other less than 90d. }

10. If both Angles at the Hypothenuse are Acute, or Obtuse, the Hypothenuse is less than a Quadrant; but if one be Acute, and the other Obtuse, the Hypothenuse is more than a Quadrant.

11. In every Triangle, greatest Sides are opposite to greatest Angles, and equal Sides to equal Angles.

12. In every Oblique Triangle, two { Acute / Obtuse } Angles being equal their opposite Sides are equal, and each { less / more } than a Quadrant.

13. If two { Acute / Obtuse } Angles are unequal, the Side opposite to the { less / greater } Angle shall be { less / more } than a Quadrant.

14. All the Angles Acute, each Side is less than a Quadrant.

These Things being premised, their Solution followeth; and first of Rectangular Spheric Triangles, in which are 16 Cases, and then Oblique-angled Spheric Triangles, in which are only 12 Cases.

Section IV. *The Solution of the* 16 *Cases of* Rectangular Spheric Triangles, *by Lord* Napier's *Catholic Proposition.*

1. IN a Rectangular Triangle, there are besides the Right-angle five Things, which the Lord *Napier* calleth the Five circular Parts of a Spheric Triangle amongst which, the Right-angle not being reckoned, the two Legs are supposed to join together.

2. Any one of these five circular Parts, may, (by Supposition) be made a middle Part, and then the two Circular Parts, which are next to that middle Part, are the Extreams Conjunct; And the other (two circular Parts) remote from that (assumed middle Part) are the Extreams Disjunct.

3. In every Case, two of the aforesaid five Circular Parts are (always) given, to find a third; of these three Things (two given and one required) one is middle Part, and the other two are extreams, either Conjunct or Disjunct.

The Parts of a Rectangular Triangle being thus distinguished, observe the Universal Proposition following, invented by the aforesaid Lord, the Inventor of Logarithms.

The Catholic Proposition.

The Sine of the middle Part and Radius, are reciprocally proportional with the Tangents of the Extreams Conjunct; and with the Sine Complements of the Extreams Disjunct: That is,

1. For Extreams Conjunct, thus;

As the Radius, is to the Tangent of one Extream; so is the Tangent of the other, to the Sine of the middle Part: And,

2. For Extreams Disjunct, thus;

As the Radius, is to the Sine Complement of one Extream, so is the Sine Complement of the other, to the Sine of the middle Part: Therefore,

Note; When the middle Part is to be found, the Radius is to be the 1st Term in the Proportion, as above; But if either of the Extreams be required, the other Extream must be the first Term: That is,

3. For Extreams Conjunct, thus;

As the Tangent of the given Extream, is to the Radius; so is the Sine of the middle Part, to the Tangent of the required Extream: And,

4. Eor Extreams Disjunct, thus;

As the Sine Complement of the given Extream, is to the Radius; so is the Sine of the middle Part, to the Sine Complement of the required Extream: But,

Note 1. That if the middle Part, or either of the Extreams Conjunct be the Hypothenuse, or either of the Oblique Angles; instead of Sine, and of Tangent, you must use the Sine Complement, and Tangent Complement.

A TABLE *of all the* Varieties *of* Extreams Conjunct and Disjunct.

Number	*Middle Part.*	*Ext. Conjunct.*	*Ext. Disjunct.*
1	Sine AB	Tang. — BC Tang. c. BAC	Sine ACB Sine AC
2	Sine c. BAC	Tang. c. AC. Tang. — AB.	Sine c. BC Sine ACB
3	Sine c. AC	Tang. c. BAC Tang. c. ACB	Sine c. BC Sine c. AB
4	Sine c. ACB	Tang. c. — C Tang. —— C	Sine BAC Sine c. AB
5	Sine BC	Tang.——AB Tang. c. ACB	Sine BAC Sine AC

2. If

2. If either of the Extreams Disjunct be the Hypothenuse, or either of the Oblique Angles, instead of Sine Complement, you must use the Sine.

And for the easier understanding these Directions; observe the foregoing Table, wherein are placed the five Circular Parts of a Rectangular Spheric Triangle, under their respective Titles, whether they may be taken for the middle Part, or for Extreams, either Conjunct or Disjunct; and unto those Parts are prefixed, Sine, Sine Complement, Tangent, and Tangent Complement; as they ought to be, and are used in the Cases following.

Prob. 1. Case 1, 2, and 3. *The Hypothenuse, and one Leg given.*

To find { 1. 2. 3. } the { Angle adjacent, Angle opposite } to the given Leg, { Other Leg }

To find	1.	the	Angle adjacent	to the given Leg.
	2.		Angle opposite	
	3.		Other Leg	

Example. *Plate* 4. *Fig.* 1.

In the Rectangular Spheric Triangle ABC, there is given,

The	Hypot. AC 54d. 25m.	ACB, BAC	required;
	Leg. BC 23d. 29m.	and AB - -	

This Triangle is made by Problem 10. of Spheric Trigonometry Geometric, in Pages 123 and 124.

1. For the contained Angle ACB, or Angle adjacent to the given Leg, the Proportion is;

As the Radius, is to the Tangent of the given Leg; so is the Tangent Complement of the Hypothenuse, to the Sine Complement of the adjacent Angle required, Or thus,

Radius .. T. Leg BC : : T. c. Hypot. AC .. S. c. Angle ACB.
T. 45d. .. T. 23d. 29m. : : T. 35d. 35m. .. S. 18d. 07m.
which subtract from 90d. rest 71d. 53m. for the Angle ACB.

Therefore the Extent (on the *Gunter*'s Scale) from Tangent of 45d. to Tangent of 23d. 29m. reacheth from Tangent 35d. 35m. to the Tangent 17d. 15m. against which, on the Line of Sines, is 18d. 07m. the 4th Term in the Proportion above.

Note; The Hyp. and given Leg { each / one } more { or —— / the other } less than a Quadrant, or 90d. the required Angle is { Acute. / Obtuse.

2. For the Angle BAC opposite to the given Leg, the Proportion is thus;

As the Sine of the Hypothenuse, is to Radius; so is the Sine of the given Leg, to the Sine of its opposite Angle: Or thus,

S. Hypot. AC .. Radius : : S. Leg BC .. S. Angle BAC.

S. 54d. 25m. .. S. 90d. : : 23d. 29m. .. Sine 29d. 20m.

Therefore the Extent (on the Gunter) from Sine 54d. 25m. to Sine of 90d. reacheth from Sine of 23d. 29m. to Sine of 29d. 20m. the fourth term in the Proportion abovesaid.

Note, The given Leg $\left\{\begin{matrix}\text{less}\\\text{more}\end{matrix}\right\}$ than a Quadrant, or 90d. $\Big\}$ the required Angle is $\left\{\begin{matrix}\text{Acute.}\\\text{Obtuse.}\end{matrix}\right.$

4. For the Leg AB, the Proportion is thus;

As the Sine Complement of the given Leg, is to Radius; so is the Sine Complement of the Hypothenuse, to the Sine Complement of the required Leg: Or thus;

S. c. Leg BC .. Radius : : S. c. Hyp. AC .. S. c. Leg AB.

S. 66d. 31m. .. S. 90d. : : S. 35d. 35m. .. S. 39d. 23m.

which subtract from 90d. resteth 50d. 37m. for the Leg AB required.

Therefore the Extent (on the Gunter's Scale) from the Sine of 66d. 31m. to Sine of 90d. will reach from Sine of 35d. 35m. to Sine of 39d. 23m. the fourth Term in the Proportion aforesaid.

Note, The Hypot. and given Leg of $\left\{\begin{matrix}\text{one ——}\\\text{a different}\end{matrix}\right\}$ Kind, the required Leg is $\left\{\begin{matrix}\text{less}\\\text{more}\end{matrix}\right\}$ than a Quadrant.

Problem 2.. Case 4, 5, and 6.

The Hypothenuse, and one Angle given;

To find $\left\{\begin{matrix}1.\\2.\\3.\end{matrix}\right\}$ the $\left\{\begin{matrix}\text{Leg opposite}\\\text{Leg adjacent}\\\text{other Angle.}\end{matrix}\right\}$ to the given Angle;

Example. *Plate* 4. *Fig.* 2.

In the Rectangular Spheric Triangle ABC, there is given; $\left\{\begin{matrix}\text{Hypot. AC 54d. 25m.}\\\text{Angle BAC 29d. 30m.}\end{matrix}\right\}$ $\left\{\begin{matrix}\text{Leg BC, AB, and}\\\text{Angle ACB required?}\end{matrix}\right.$

This Triangle is made by Problem 11. of Spheric Trigonometry Geometric, in Pages 124 and 125.

1. For the Leg BC, opposite to the given Angle, the Proportion is thus;

As Radius, is to the Sine of the Hypothenuse; so is the Sine of the given Angle, to the Sine of its opposite Leg required: Or thus.

Radius

Radius .. S. Hypot. AC : : S. Angle BAC .. S. Leg BC.
S. 90d. .. S. 54d. 25m. : : S. 29d. 30m. .. S. 23d. 36m.

Note; the given Angle { Acute, / Obtuse, } the required Leg is { less / more } than a Quadrant.

2. For the Leg AB, adjacent to the given Angle, the Proportion is this;

As the Tangent Complement of the Hypothenuse, is to Radius; so is the Sine Complement of the given Angle to the Tangent of its adjacent Leg: Or,

As Radius, is to the Sine Complement of the given Angle; so is the Tangent of the Hypothenuse, to the Tangent of the Leg required: That is,

Radius .. S. c. BAC : : T. Hypot. AC .. T. Leg AB.
S. 90d. .. S. 60d. 30m. : : T. 54d. 25m. .. T. 50d. 34m.

Note; When each given thing is less, or more than 90d. the required Leg is less than a Quadrant; but if one is less, and the other more, the required Leg is more than a Quadrant.

3. For the Angle ACB, the Proportion is this;

As the Tangent Complement of the given Angle, is to Radius; so is the Sine Complement of the Hypothenuse, to the Tangent Complement of the other Angle; Or thus,

As Radius is to the Sine Complement of the Hypothenuse; so is the Tangent of the given Angle, to the Tangent Complement of the Angle required: That is,

Radius .. S. c. Hypot. AC : : T. Angle BAC .. T. c. Angle ACB.
S. 90d. .. S. 35d. 35m. : : T. 29m. 30m. .. T. 18d. 13m.
which subtract from 90d. rests 71d. 47m. for the Angle ACB.

Note; When each given Thing is less, or more than 90d. the required Angle is Acute; but when one is more, and the other less, 'tis Obtuse.

Problem 3. Case 7, 8, 9. *A Leg and its adjacent Angle given*;

To find { 1. / 2. / 3. } the { Leg / Angle / Hypothenuse } opposite to the given { Angle. / Leg.

Example. *Plate* 4. *Fig*. 3.

In the Rectangle Triangle ABC, Right-angled at B, there is given { Leg — AB 50d. 30m. / Angle BAC 29d. 30m. } Leg BC, Angle ACB. and Hypot. AC required?

This

This Triangle is made by Problem 12, of Spheric Trigonometry Geometric, in Pages 125 and 126.

1. For the Leg BC opposite to the given Angle, the Proportion is,

As the Tangent Complement of the given Angle, is to the Radius; so is the Sine of its adjacent Leg, to the Tangent of its opposite Leg required: Or thus;

As the Radius is to the Sine of the given Leg; so is the Tangent of its adjacent Angle, to the Tangent of its opposite Leg; That is,

Radius .. S. Leg AB : : T. Angle BAC .. T. Leg BC.
S. 90d. .. S. 50d. 30m. : : T. 29d. 30m. .. T. 23d. 35m.

Note; The given Angle Acute, the required Leg is less than a Quadrant, but when Obtuse, then more than a Quadrant.

2. For the Angle ACB, opposite to the given Leg, the Proportion is;

As Radius, is to the Sine Complement of the given Leg; so is the Sine of its adjacent Angle, to the Sine Complement of its opposite Angle required; That is,

Radius .. S. c. Leg AB : : S. Angle BAC .. S. c. Angle ACB.
S. 90d. .. S. 39d. 30m. : : S. 29d. 30m. .. S. 18d. 15m.
whose Complement 71d. 45m. is the Angle ACB.

Note; The given Leg less, or more than 90d. the Angle required is Acute, or Obtuse accordingly.

3. For the Hypothenuse, the Proportion is;

As the Tangent of the given Leg, is to the Radius; so is the Sine Complement of its adjacent Angle, to the Tangent Complement of the required Hypothenuse: Or thus,

As the Radius, is to the Sine Complement of the given Angle; so is the Tangent Complement of its adjacent Leg, to the Tangent Complement of the Hypothenuse; That is,

Radius .. S. c. Angle BAC : : T. c. Leg AB .. T. c. Hyp. AC.
S. 90d. .. S. 60d. 30m. : : T. 39d. 30m. .. T. 35d. 40m.
whose Complement 54d. 20m. is the Hypothenuse AC.

Note; When each Thing is less, or more than 90d. the Hypothenuse is less than a Quadrant; but if one is less, and the other more, then the Hypothenuse is more than a Quadrant.

Prob. 4. Case 10, 11, 12. *A Leg and its opposite Angle given.*

To find { 1. Leg / 2. Angle } adjacent to the given { Angle / Leg. }
3. Hypothenuse.

Example.

Example. *Plate* 4. *Fig.* 4.

In the Rectangular Spheric Triangle ABC, there is,

given { Leg — BC 23d. 29m. / Angle BAC 29d. 30m. } Leg AB, Angle ACB, and Hypothenuse AC required.

This Triangle is made by Problem 13, of Spheric Trigonometry Geometric, in Page 126.

1. For the Leg AB, adjacent to the given Angle, the Proportion is this;

As Radius, is to the Tangent of the given Leg; so is the Tangent Complement of its opposite Angle, to the Sine of the other Leg required; That is,

Radius ·· T. Leg BC : : T. c. Angle BAC ·· S. Leg AB.

T. 45d. ·· T. 23d. 29m. : : T. 60d. 30m. ·· S. 50d. 10m.

Note; If the unknown Angle be Acute, or Obtuse, the required Leg accordingly is less, or more than a Quadrant.

Or, the Hypothenuse and given Leg, each less, or more than 90d. the required Leg is less than a Quadrant; but if one is more, and the other less, 'tis more than a Quadrant.

2. For the Angle ACB adjacent to the given Leg, the Proportion is this;

As the Sine Complement of the given Leg, is to the Radius; so is the Sine Complement of its opposite Angle, to the Sine of its adjacent Angle: That is,

S. c. Leg BC ·· Radius : : S. c. Angle BAC ·· S. Angle ACB.

S. 66d. 31m. ·· S. 90d. : : S. 60d. 30m. ·· S. 71d. 37m.

Note; If the Hypothenuse and given Angle be each less, or more than 90d. the Angle required is Acute, but when one is more, and the other less, 'tis Obtuse.

Or, the Leg adjacent to the given Angle being less, or more than 90d. accordingly the Angle required is Acute or Obtuse.

3. For the Hypothenuse AC, the Proportion is this:

As the Sine of the given Angle, is to the Radius; so is the Sine of its opposite Leg, to the Sine of the Hypothenuse required: That is,

S. Angle BAC ·· Radius : : S. Leg BC ·· S. Hypothenuse AC.

S. 29d. 30m. ·· S. 90d. : : S. 23d. 29m. ·· S. 54d. 01m.

Note; If both Legs, or both Angles are of one Kind, the Hypothenuse is less than a Quadrant, but if they be of different Kinds, the Hypothenuse is more than a Quadrant.

Problem 5. Case 13 and 14. *Both the Legs given; to find 1. Either of the Angles? 2. The Hypothenuse?*

Example. *Plate* 4. *Fig.* 5.

In the Rectangular Spheric Triangle ABC, there is given, Leg { AB 50d. 30m. / BC 23d. 29m. } Angle BAC, or ACB, and Hypothenuſe AC required.

This Triangle is made by Prob. 14. of Spheric Trigonometry Geometric, in Page 125.

1. For either of the Angles, the Proportion is,

As the Tangent of one Leg, is to Radius; ſo is the Sine of the other Leg, to the Tangent Complement of the Angle oppoſite to the firſt Leg; Or thus,

As Radius, is to the Sine of one Leg; ſo is the Tangent Complement of the other Leg, to the Tangent Complement of the Angle oppoſite to the laſt Leg; That is,

Radius .. S. Leg AB : : T. c. Leg BC .. T. c. Angle BAC.

S. 90d. .. S. 50d. 30m. : : T. 66d. 31m. .. T. 60d. 37m. whoſe Complement 29d. 23m. is the Angle BAC: Alſo it is,

Radius .. S. Leg BC : : T. c. Leg AB .. T. c. Angle ACB.

S. 90d. .. S. 23d. 29m. : : T. 39m. 30m. .. T. 18d. 11m. whoſe Complement 71d. 49m. is the Angle ACB.

Note, If the Leg oppoſite to the required Angle be leſs than a Quadrant, the Angle ſought is Acute; but if greater, than 'tis Obtuſe.

2. For the Hypothenuſe, the Proportion is thus;

As Radius, is to the Sine Complement of one Leg; ſo is the Sine Complement of the other Leg, to the Sine Complement of the Hypothenuſe; That is,

Radius .. S. c. Leg AB : : S. c. Leg BC .. S. c. Hypot. AC.

S. 90d. .. S. 39d. 30m. : : S. 66d. 31m. .. S. 35d. 41m. whoſe Complement 54d. 19m. is the Hypothenuſe AC.

Note, If both Legs be of one Kind, the Hypothenuſe is leſs than a Quadrant; but when of different Kinds (that is, one Leg more, the other leſs than a Quadrant) then the Hypothenuſe is more than a Quadrant.

Problem 6. Caſe 15 and 16. *Both the Oblique Angles given; To find,* 1. *Either of the Legs?* 2. *The Hypothenuſe?*

Example. *Plate* 4. *Fig.* 6.

In the Rectangular Spheric Triangle ABC, is given,

Angle { BAC 29d. 30m. / ACB 71d. 56m. } Leg AB, or BC, and Hypothenuſe AC required?

This Triangle is made by Problem 15, of Spheric Trigonometry Geometric, in Page 128.

1. For either of the Legs, the Proportion is thus;

As

As the Sine of one Angle, is to Radius; ſo the Sine Complement of the other Angle, to the Sine Complement of its oppoſite Leg; That is,

S. Angle BAC .. Radius : : S. c. Angle ACB .. S. c. Leg AB.
S. 29d. 30m. .. S. 90d. : : S. 18d. 04m. .. S. 39d. 02m.
whoſe Complement 50d. 58m. is the Leg AB: Alſo it is,
S. Angle ACB .. Radius : : S. c. Angle BAC .. S. c. Leg BC.
S. 71d. 56m. .. S. 90d. : : S. 60d. 30m. .. S. 66d. 17m.
whoſe Complement 23d. 43m. is the Leg BC.

Note; If the Angle oppoſite to the required Leg be Acute, the Leg ſought is leſs than a Quadrant; but if Obtuſe, then 'tis greater than a Quadrant.

2. For the Hypothenuſe, the Proportion is thus;

As Radius, is to the Tangent Complement of one Angle; ſo is the Tangent Complement of the other Angle, to the Sine Complement of the Hypothenuſe; That is,

Radius .. T. c. Angle BAC : : T. c. Angle ACB .. Hyp. AC.
T. 45d. .. T. 60d. 30m. : : T. 18d. 04m. .. S. 35d. 13m.
whoſe Complement 54d. 47m. is the Hypothenuſe AC.

Note; The Angles of one Kind, the Hypothenuſe is leſs than a Quadrant; but when of different Kinds, 'tis more than a Quadrant.

Section V. *Four Axioms, by which the 12 Caſes of Oblique-angled Spheric Triangles are reſolved.*

Axiom 1. IN all Spheric Triangles, the Sines of their Sides are in direct Proportion to the Sines of their oppoſite Angles; That is,

1. As the Sine of a Side, is to the Sine of its oppoſite Angle; ſo is the Sine of any other Side, to the Sine of its oppoſite Angle.

2. As the Sine of an Angle is to the Sine of its oppoſite Side; ſo is the Sine of any other Angle, to the Sine of its oppoſite Side.

Axiom 2. *Firſt*, As the Sine of half the Sum of two Sides, (containing an Angle) is to the Sine of half their Difference;

So is the Tangent Complement of half the contained Angle, to the Tangent of half the Difference of the other two Angles.

Again,

Secondly, As the Sine Complement of half the Sum of two Sides (containing an Angle) is to the Sine Complement of Half their Difference;

So is the Tangent Complement of half the contained Angle, to the Tangent of half the Sum of the other two Angles.

Axiom 3. *Firſt*, As the Sine of half the Sum of two Angles; is to the Sine of half their Difference; So

So is the Tangent of half their interjacent Side, to the Tangent of half the Difference of the other two Sides: Again,

Secondly, As the Sine Complement of half the Sum of two Angles, is to the Sine Complement of half their Difference;

So is the Tangent of half their interjacent Side, to the Tangent of half the Sum of the other two Sides.

Axiom 4. As the Rectangle of the Sines of the two containing Sides, is to the Square of the Radius;

So is the Rectangle of the Sine of half the Sum of the three Sides, and of the Sine of the Difference between the said half Sum and the Side opposite the Angle, to the Square of the Sine Complement of half the contained Angle sought.

The Explanation and Use of these four Axioms, you will find in the Solution of the following Cases of Oblique Spheric Triangles.

Problem 7. Case 1, 2 and 3.

Two Sides, and an opposite Angle given; to find

1. The Angle opposite to the other Side,
2. The Angle contained between them,
3. The third Side ———————

} if the required opposite Angle be foreknown to be Acute, or Obtuse.

Example. *Plate* 4. *Fig*. 7.

In the Oblique Spheric Triangle ACD, there is

given	Side---- AC 34d. 07m.	Angle ACD being Obtuse	req.
	Side---- AD 65d. 20m.	Angle CAD ———	
	Angle ADC 27d. 30m.	Side CD ———	

This Triangle is made by Prob. 16, of Spheric Trigonometry Geometric, in Pages 128 and 129.

1. For the opposite Angle ACD, the Proportion (by Axiom 1.) is thus;

As the Sine of the Side AC, is to the Sine of the Angle ADC; So is the Sine of the Side AD, to the Sine of the Angle ACD; That is,

S. Side AC ·· S. Angle ADC : : S. Side AD ·· S. Angle ACD.
S. 34d. 07m. ·· S. 27d. 30m. : : S. 65d. 20m. ·· S. 48d. 25m.

Which subtract from 180 resteth 131d. 35m. for the Angle ACD, it being required to be Obtuse.

2. For the contained Angle CAD (having before found the opposite Angle) the Proportion (by Axiom 2. Part 1. inverted,) is thus;

As the Sine of half the Difference of the Sides AD and AC, is to the Sine of half their Sum; so is the Tangent of half the Difference of the Angles ACD, and ADC, to the Tangent Complement of half the Angle CAD; and,

3. For

3. For the third Side CD, (having found the opposite Angle) the Proportion, (by Axiom 3, Part 1, inverted) is thus;

As the Sine of half the Difference of the Angles ACD, and ADC, is to the Sine of half their Sum;

So is the Tangent of half the Difference of the Sides AD and AC, to the Tangent of half the Side CD: That is,

	d. m.		d. m.		d. m.		m.
Side AD	65.20			Ang. ACD	131.35		
Side AC	34.07			Ang. ADC	27.30		
Sum is	99.27	½ Sum	49.43	Sum is	159.05	½ Sum	79.32
Differ.	31.13	½ Diff.	15.36	Diff. is	104.05	½ Diff.	52.02

S. ½ Diff. Sides .. S. ½ their Sum : : T. ½ Diff. Ang. .. T. c. ½ CAD.
S. 15d. 36m. .. S. 49d. 43m. : : T. 52d. 02m. .. T. 74d. 37m.

Which subtract from - - - - - - - - - - 90d. 00m.

Remainder is half the Angle CAD - - - - 15d. 23m.

Which being doubled - - - - - - - - - 15d. 23m.

Produceth the required Angle CAD - - - - 30d. 46m.

S. ½ Diff. Ang. .. S. ½ their Sum : : T. ½ Diff. Sides .. T. ½ CD.
S. 52d. 02m. .. S. 79d. 32m. : : T. 15d. 36m. .. T. 19d. 12m.

Which being doubled - - - - - - - - - 19d. 12m.

Produceth the required Side CD - - - - - 38d. 24m.

Problem 8. Case 4, 5, and 6.

Two Angles, and one Side opposite given; to find,

1. The Side opposite to the other Angle,
2. The interjacent, or Side between them
3. The third Angle ——— ———

} if the required opposite Side be foreknown to be more, or less than a Quadrant.

Example, *Plate* 4. *Fig.* 8.

In the Oblique Spheric Triangle ACD, *there is given,*

Angle CAD 30d. 48m.	Side——AC less than 90d.
Angle ADC 27d. 30m.	Side——AD ——— —
Side — CD 38d. 28m.	Angle ACD ——— —

} required.

This Triangle is made by Problem 17. of Spheric Trigonometry Geometric, in Page 129.

1. For the opposite Side AC, the Proportion by Ax. 1. is thus;

As the Sine of the Angle CAD, is to the Sine of the Side CD; so is the Sine of the Angle ADC, to the Sine of the Side AC; That is,

S. Angle CAD .. S. Side CD : : S. Angle ADC .. S. Side AC req.
S. 30d. 48m. .. S. 38d. 28m. : : S. 27d. 30m. .. S. 34d. 07m.

2. For the interjacent Side AD, (having first found the opposite Side) the Proportion (by Axiom 3. Part 1. in

As the Sine of half the Difference of the Angles CAD and ADC, is to the Sine of half their Sum;

So is the Tangent of half the Difference of the Sides CD and AC, to the Tangent of half the Side AD: And,

3. For the third Angle ACD, (having firſt found the oppoſite Side) the Proportion, (by Axiom 2. Part 1. inverted) is thus;

As the Sine of half the Difference of the Sides CD and AC, is to the Sine of half their Sum;

So is the Tangent of half the Difference of the Angles CAD and ADC, to the Tangent Complement of half the Angle ACD; That is,

	d. m.				d. m.		d. m.
Ang. { CAD	30 48			Side { CD	38 28		
Ang. { ADC	27 30			Side { AC	24 07		
Sum is	58 18	½ {	Sum 29.09	Sum is	72 35	½ {	Sum 36 17
Difference	03 18		Diff. 1.39	Diff. is	4 21		Diff. 02 10

S. half Diff. Ang. .. S. half their Sum :: T. ½ Diff. Sides .. T. ½ AD
S. 01d. 39m. .. S. 29d. 09m. :: T. 02d. 10m. .. T. 32d. 37m.
Which being doubled ——— ——— 32d. 37m.

Produceth the Interjacent Side AD ——— 65d. 14m.

S. ½ Diff. Sides .. S. ½ their Sum :: T. ½ Diff. Angles .. T.c. ½ ACD
S. 02. 10m. .. S. 36d. 17m. :: T. 01d. 39m. .. T. 24d. 16m.
Which ſubtract from ——— ——— — 90d. 00m.

Remainder is half the Angle ACD ——— 65d. 44m.
Which being doubled ——— ——— 65d. 44m.

Produceth the Angle ACD to be ——— ——— 131d. 28m.

Note; In the ſix preceding Caſes, the three given Terms without the Quality of a Fourth, are not ſufficient, whereby one ſingle Anſwer may be found; and the Quality of the Fourth is not always diſcoverable by the given Terms; Therefore they are called the Six doubtful Caſes.

Problem 9. Caſe 7, and 8.

Two Sides, and one Angle between them given; to find

1. Either of the other Angles;
2. The third Side, or Side oppoſite to the given Angle.

Example. *Plate* 4. *Fig.* 9.

In the Oblique Spheric Triangle ACD, there is

given {	Side - - AC 34d. 06m.	Angle ACD, or	} required;
	Side - - AD 65d. 20m.	Angle ADC, and	
	Angle CAD 30d. 46m.	Side CD; is	

This

This Triangle is made by *Problem* 18. of *Spheric Trigonometry Geometric*, in Pages 129 and 130.

For the Angle ACD, or ADC, the Proportion (by *Axiom* 2. *Parts* 1 and 2.) is thus;

		d. m.		d. m.		
Side	AD	65.20	Angle CAD	30.46		d. m.
	AC	34.06	The Half is	15.23	- - - - - -	74.37
Sum of Sides		99.26	The ½ Sum.	49.43	its Comp.	40.17
Differ. Sides		31.14	The ½ Diff.	15.37	- - - - - -	74.13

As S. ½ Sum AD & AC .. S. ½ Diff. : : T. c. ½ CAD .. T. ½ Sum Ang.
As S. 49d. 43m. .. S. 15d. 37m. : : T. 74d. 37m. .. T. 52d. 03m.

Then again,

As S. c. ½ Sum Sides .. S. c. ½ Diff. : : T. c. ½ CAD : T. ½ Sum Ang.
As S. 40d. 17m. .. S. 74d. 23m. : : T. 74d. 37m. .. T. 79d. 32m.
To it add the Half Difference found above - - - - 52d. 03m.
The Sum is the greater Angle ACD - - - - - 131d. 35m.
And being subtracted, is the less Angle ADC - - 27d. 29m.

Note; If the Sum of the two containing Sides exceeds a Semicircle, then subtract each Side from 180 Degrees, and proceed with those Remainders as with the Sides given; the Proportion produces the Supplement of the Angle required to a Semicircle.

For the third Side CD, or Side opposite to the given Angle, the Proportion (after the Work above is done) may be made by *Axiom* 1. Or it may be deduced from the Lord *Napier's Catholic* Proposition (by a supposed Perpendicular, let fall from one End of the less given Side, to the greater Side given, reducing the Oblique Triangle into two Rectangulars) which finds the Side opposite to the given Angles at two Proportions, without finding the Angles; And it is thus,

1. As Radius is to the Sine Complement of the contained Angle; So is the Tangent of the less given Side, to the Tangent of a Fourth Arc.

Then, if the contained Angle be Acute, subtract the Fourth Arc from the greater given Side, but when it is obtuse from the Supplement thereof to a Semi-circle, or 180 Degrees, the Remainder is called the *Residual Arc*; Then,

2. As the Sine Complement of the Fourth Arc, is to the Sine Complement of the *Residual Arc*;

So is the Sine Complement of the less given Side, to the Sine Complement of the Sine required; That is,

As Radius .. S. c. Angle CAD : : T. Side AC .. T. Fourth Arc.
As S. 90d. .. S. 59d. 14m. : : S. 34d. 06m. .. T. 30d. 11m.
which being subtracted from the Side AD - - 65d. 20m.

The Remainder is the Residual Arc - - - - - 35d. 09m.

Then,

As S.c. 4th Arc. .. S.c. Resid. Arc. : : S.c. Side AC. .. S.c. Side CD.
As S. 59d. 49m. .. S. 54d. 51m. : : 55d. 54m. .. S. 51d. 34m.
whose Complement 38d. 26m. is the Side CD required.

Note; When the contained Angle, and Residual Arc, are each more, or less than 90 Degrees, the Side sought is less than a Quadrant; but when one is more, and the other less, 'tis more than a Quadrant.

Problem 10. Case 9 *and* 10.

Two Angles and one Side between them given;

To find { 1. *Either of the other Sides?*
2. *The Angle opposite to the given Side?* }

Example. In the *Oblique Spheric Triangle* ACD, there is given,

The	Angle ACD 131d. 34m.	Side	AC, or	required.
	Angle ADC 27d. 30m.	Side	AD, and	
	Side CD 21d. 28m.	Angle	CAD is	

This Triangle is made by *Problem* 19. of *Spheric Trigonometry Geometric*, in Page 130. See *Plate* 4. *Fig.* 10.

For the Side AC or AD, the Proportion by *Axiom* 3. *Part* 1. and 2.) is thus;

	d. m.		d. m.			d. m.
Angle ACD	131. 34.	The Side CD	38. 28.			
Angle ADC	27. 30.	The Half is	19. 14.			
Sum of Angles	159. 04.	The ½ Sum	79. 32	its Com.		10. 28
Difference is	104. 04.	The ½ Diff.	52. 02	its Com.		37. 58

As S. ½ Sum Ang. .. S. ½ their Diff. : : T. ½ Side CD .. T. ½ Diff. Sides
As S. 79d. 32m. .. S. 52d. 02m. : : T. 19d. 14m. : T. 15d. 38m.

Then again,

As S.c. ½ Sum An. .. S.c. ½ their Diff. : : T. ½ Side CD. .. T. ½ Sum Sides
As S. 10d. 28m. .. S. 37d. 58m. : : T. 19d. 14m. : T. 49d. 45m.
To it add the half Difference found above - - - - 15d. 38m.

The Sum is the greater Side AD - - - - - - - - - 65d. 23m.

And being subtracted, is the less Side AC - - - - 34d. 07m.

Note; If the Sum of the two given Angles exceed 180 Degrees then subtract each given Angle from 180 Degrees, and proceed with those Remainders as with the Angles given; the Operation will produce each required Side's Supplement to a Semicircle, or 180 Degrees.

For

For the Angle CAD, the Angle opposite to the given Side, the Proportion (after the Work is done) may be by *Axiom* 1. Or from the *Catholic Proposition* (by a perpendicular supposed to be let fall from the greater given Angle to its opposite Side) thus;

As Radius is to the Sine Complement of the interjacent Side; so is the Tangent of the less given Angle, to the Tangent of a Fourth Arc.

Then, if the interjacent Side be more than a Quadrant, subtract the Fourth Arc from the greater given Angle; but when 'tis less, from the Supplement thereof to 180 Degrees, the Remainder is the Residual Arc; Then,

2. As the Sine Complement of the Fourth Arc, is to the Sine Complement of the Residual Arc;

So is the Sine Complement of the less given Angle, to the Sine Complement of the Angle required. That is,

As Radius .. S. c. Side CD : : T. Angle ADC .. T. 4th Arc.

As S. 90d. .. S. 51d. 32m. : : T. 27d. 30m. .. T. 24d. 22m.

which subtract from (Suppl. of the Angle ACD) 48d. 26m.

The Remainder is the Residual Arc - - - - - 26d. 15m.

Then,

S.c.4th Arc .. S. c. Resid. Arc. : : S.c.An.ADC .. S.c. An CAD req.

S.67d.49m. .. S. 63d. 45m. : : S. 62d. 30m. .. S. 59d. 13m.

whose Complement 30d. 47m. is the Angle CAD required.

Note; When the adjacent Side and Residual Arc are each more or less than 90d. the Angle sought is Acute; but when one is more, and the other less, 'tis Obtuse.

Problem 11. Case 11. *Three Sides given, to find any of the Angles.*

Example. *Plate* 4. *Fig.* 11.

In the Oblique Spheric Triangle ACD there is given;

The Side	AD 65d. 20m. CD 38d. 26m. AD 34d. 08m.	Angle	ACD, or ADC, or CAD required?

This Triangle is made by Problem 20, of Spheric Trigonometry Geometric, in Pages 130 and 131.

The Resolution of this, and the following Case depends upon the 4th Axiom; and for the greater Dispatch, observe the following Directions, *viz.*

1. Add the three Sides together, and from their half Sum subtract the Side opposite to the Angle required, nothing the Remainder.

2. To the Complement Arithmetic of the Logarithmic Sines of the containing Sides, add the Logarithmic Sines of the half Sum and the Remainder; Half the Total of these four Logarithms, is the Logarithmic Sine of Half the required Angle's Supplement to 180 Degrees.

The Operation for the Angle ADC, is thus;

		d. m.		
Side	DC	38 26	The containing Sides	S. co. ar. 0.206487
	AC	34 08		S. co. ar. 0.250944
	AD	65 20	½ Sum Sides 68d. 57m.	S. co. ar. 9.970006
Sum is	-	137 54	Remainder is 3d. 37m.	S. - - 8.799897
Half Sum is		68 57	Sum of the four Logarithms is	19.227334
Remain	-	3 37	24d. 15m. S. ½ Sum	9.613667

Double it - - - 24d. 14m.

The Sum is - - 48d. 30m.

Which subtract from 180d. 00m.

Remainder is the Angle ACD 131d. 30m.

In like Manner you may find any other Angle by Logarithms.

Or thus,

By Gunter's Scale, *say*,

1. As Radius, is to the Sine of one of the containing Sides; So is the Sine of the other containing Side, to a Fourth Sine. Then,

2. As that 4th Sine, is to the Sine of the half Sum of the three Sides; so is the Sine of the Remainder to a fifth Sine; against which on the Line of versed Sines is the Angle required.

That is,

As Radius .. S. Side CD : : S. Side AC .. a Fourth Sine.
As S. 90d. .. S. 38d. 26m. : : S. 34d. 08m. .. S. 20d. 25m. Then,
As Fourth Sine .. S. ½ Sum Sides : : S. Remainder .. a Fifth Sine.
As S. 20d. 25m. .. S. 68d. 57m. : : S. 03d. 37m. .. S. 9d. 43m.
against which on the Line of the versed Sines, is 131d. 30m. the Angle ACD as before.

Problem 12. Case 12. *The Angles given; to find a Side.*

Example. *Plate* 4. *Fig.* 12.

In the Oblique Spheric Triangle ACD; there is given,

The Angle	ACD	30d. 47m.	: The Side	CD, or
	ADC	27d. 30m.		AC, or
	CAD	131d. 34m.		AD, required?

This Triangle is made by Problem 21. of *Spheric Trigonometry Geometric*, in Page 131.

This

This Case is likewise perform'd by the Directions in Case 11, the Angles being accounted Sides, and the Sides Angles; and then taking the Supplement of the greater (given) Angle to 180 Degrees.

For the Side AD, the Operation is thus;

		d. m.		
Sup. Ang.	CAD	48 26	The adjacent Angles	S. co. ar. 0.125992
Angle	ADC	27 30		S. co. ar. 0.335594
	ACD	30 47	½ Sum Ang. 53d. 21m. S. - - -	9.904335
Sum is -		[illegible]6.43	Remainder 22d. 34m. S. - - -	9.584058
Half Sum is -		53 21	Sum of the four Logarithms is	19.949979
Remainder -		22 34	- - - - - 70.44 S. half Sum	9.974989

Double it - - - - - - 70.44
The Sum is - - - - 141.28 Which
Subtract from - - - 180.00
Remainder is the Side AD 38.32

In like manner may any other Side be found by Logarithms; Or thus,

By Gunter's *Scale*.

1. As Radius, to the Sine of one of the adjacent Angles (to the Side required) so is the Sine of the other adjacent Angles, to a Fourth Side.
2. Then, as the Fourth Sine, is to the Sine of the half Sum of the three Angles; so is the Sine of the Remainder, to a fifth Sine; against which, on the Line of versed Sines, is the Side required.

That is,
As Radius .. S. Sup. CAD : : S. Angle ADC : a Fourth Sine.
As S. 90d. .. S. 48d. 26m. : : S. 27d. 30m. .. S. 20d. 13m.

Then again,
As Fourth Sine .. S. ½ Sum Angles : : S. Remainder .. a Fifth Sine.
As S. 20d. 13m. .. S. 53d. 21m. : : S. 22d. 34m. .. 65d. 00m.
Against which, on the Line of versed Sines, is - - 38d. 32m. the Side AD, as above.

Note; When the greatest Side (CD, which ever is opposite to the greatest Angle) is required, the Operation will produce the Supplement thereof to a Semi-Circle; wherefore, if it be subtracted from 180d. it leaves the Side sought: Or (by the

2. To the Complement Arithmetic of the Lo[...] of the containing Sides, add the Logarithmic Si[...] Sum and the Remainder; Half the Total of th[...] rithms, is the Logarithmic Sine of Half the requir[...] plement to 180 Degrees.

The Operation for the Angle ADC, is [...]

		d. m.		
Side	DC	38 26	The containing Sides	S. co[...]
	AC	34 08		S. co[...]
	AD	65 20	½ Sum Sides 68d. 57m.	S. co[...]
Sum is -		137 54	Remainder is 3d. 37m.	S. [...]
Half Sum is		68 57	Sum of the four Logarithm[...]	
Remain -		3 37	24d. 15m.	S. ½ Su[...]

Double it - - - 24d. 14m.

The Sum is - - 48d. 30m.

Which subtract from 180d. 00m.

Remainder is the Angle ACD 131d. 30m.

In like Manner you may find any other Angle b[...]

Or thus,

By Gunter's Scale, *say*,

1. As Radius, is to the Sine of one of the contai[...] So is the Sine of the other containing Side, to a [...] Then,

2. As that 4th Sine, is to the Sine of the half Su[...] Sides; so is the Sine of the Remainder to a fifth [...] which on the Line of versed Sines is the Angle req[...]

That is,

As Radius .. S. Side CD : : S. Side AC .. a Fourth [...]

As S. 90d. .. S. 38d. 26m. : : S. 34d. 08m. .. S. 20d. 2[...]

As Fourth Sine .. S. ½ Sum Sides : : S. Remainder ..[...]

As S. 20d. 25m. .. S. 68d. 57m. : : S. 03d. 37m. ..[...] against which on the Line of the versed Sines, is 13[...] Angle ACD as before.

Problem 12. Case 12. *The Angles given; to fi[...]*

Example. *Plate* 4. *Fig.* 12.

In the Oblique Spheric Triangle ACD; there i[...]

The Angle	ACD	30d. 47m.		CD,
	ADC	27d. 30m.	[...]	AC[...]
	CAD	131d. 34m.		

This Triangle is made by Pr[...] *Geometric*, in Page 131.

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Logarithms) if you omit this Part of the Operation, (viz. the Subtraction from 180 Degrees) you have the Side required.

So much for the Doctrine of Spheric Triangles; the Application follows.

CHAPTER VI.

Containing the Description and Use of the Globes.

HAving finished Spheric Trigonometry, it remains to shew its Uses in Geography, Great Circle Sailing, and Astronomy. In order to the perfect Understanding of these, it is requisite first to be acquainted with the Nature and Use of the Globes, which I will endeavour to perform with Brevity, and as clearly as possible.

Section I. *The Description and Use of Globes in General.*

Definition 1. A Globe (as to its Name and Figure) is so generally known, that 'tis needless here to produce a Mathematical Definition of it; but as to the Kinds of Material Globes, and their Parts, it is necessary something should be said.

2. The Kinds of Globes are two; Terrestrial and Cœlestial, which, together with their Appurtenances, are for the lively Representation of the Natural Situation, and Position of the Earth and Heavens, and are useful for the demonstrating (by Representation and Resolution of) any Problem belonging to the Sphere of Heaven, or Earth, either in Geography, Navigation, or Astronomy.

3. The Appurtenances appertaining to Material Globes are Six; the Body or Globe itself, Brazen Meridian, Quadrant of Altitude and its Screw, Hour-Circle and Index, Wooden Frame or Horizon, and Brass Semi-Circle of Position; all may be used with either Globe; yet the last being of least Use, may be supplied by the Quadrant of Altitude.

4. The Body, or Globe itself, is an Emblem of Heaven, or of Earth; on which are drawn diverse Lines and Circles, useful and proper for the Explication thereof.

5. The Brazen Meridian, is the Ring in which the Globe hangeth, and turneth upon its Axis, being two Wires issuing from its Body; and is divided into four Quadrants, each 90 Degrees, and figured 10, 20, 30, &c. to 90.

6. The